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SEPARATION OF MIXTURES IN CONNECTED CHANNELS

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The stationary modes of thermal convection of a binary mixture in connected channels of finite height were studied experimentally and theoretically. The effects of positive and negative thermal diffusion on the convection were examined. The ranges of parameters corresponding to the modes of soft and rigid initiation of convection were determined. Vertical distributions of the temperature and concentration fields were found for various values of the thermal diffusion parameter.

Key words: *binary fluid, connected channels, positive and negative thermal diffusion effects, separation of mixture into components.*

Introduction. As is known, thermoconcentration convection occurs in a nonuniformly heated fluid mixture of complex composition in a gravity filed. In this case, density inhomogeneity arises from a competition of two convection mechanisms. The first mechanism has a thermogravitation nature and is due to the temperature dependence of the fluid density, and the second mechanism is due to the inhomogeneous concentration distribution of the heavy or light component in the mixture. The existence of two competing lifting forces leads to more complex behavior of the system than in the case of a homogeneous fluid. The forces are determined by the temperature and concentration coefficients of the density and by the temperature and concentration gradients. In most mixtures, the presence of temperature gradients is responsible for the occurrence of concentration gradients. (This phenomenon is called thermal diffusion.) In this case, free convective flow is influenced by the concentration inhomogeneity due to thermal diffusion.

A horizontal layer of a binary mixture heated from below undergoes thermal diffusion separation along the vertical, which, in the case of a positive Soret effect with equilibrium instability promotes the development of monotonic perturbations [1]. In the case of anomalous thermal diffusion, where thermogravitation and anomalous thermal diffusion mechanisms of convection initiation counteract with each other, oscillatory mechanisms of development of equilibrium instability dominate.

The purpose of the present work was to study stationary and oscillatory convective flows of a binary mixture in connected channels of finite height taking into account positive and negative thermal diffusion. Convection of one-component fluids and binary mixtures in connected channels has been studied previously [2, 3]. In this case, thermal diffusion was ignored, and the source of concentration inhomogeneities was a linear vertical concentration profile of the heavy component on the boundaries of the channels.

1. Formulation of the Problem. We consider two vertical channels of finite height with ideal heat-conducting boundaries (Fig. 1). The channels have square section and are connected at the top and bottom by short bridges of the same profile. In the theoretical study, the influence of the bridges is ignored. The calculations are performed for the same set of geometrical parameters as in the experiment. Estimates show that, in this case, the thermal interaction of the left and right channels can be ignored. The coordinate system was choose so that the z axis was directed upward along the channels. In this coordinate system, $\boldsymbol{\gamma}(0, 0, 1)$ is a unit vector directed vertically upward. The convective loop is heated from below so that a linear temperature distribution is maintained on the vertical boundaries of the channels. Below, it will be shown that, at such a temperature distribution makes possible the state of mechanical equilibrium in the binary fluid [4, 5].

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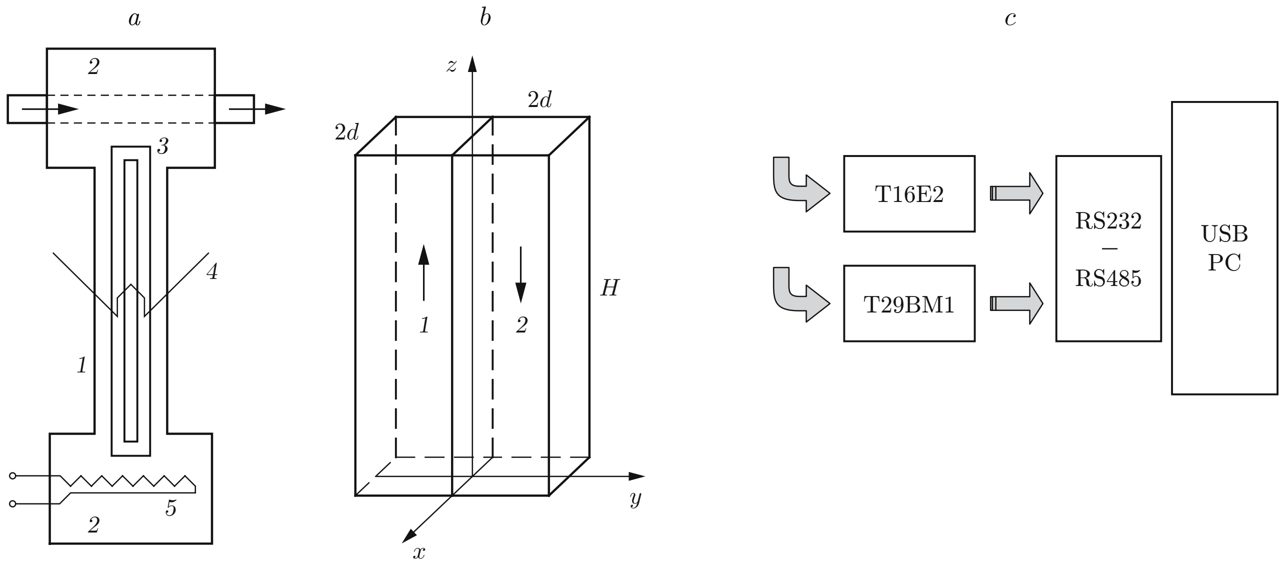

Fig. 1. Diagrams of experiment: (a) diagram of experimental setup [copper rod (1), heat-transfer device (2), connected channels (3), thermocouple for recording convective flows (4), electrical heater (5)]; (b) diagram of channels and attached coordinate system; (c) diagram of thermocouple recording.

2. Mathematical Model. In describing convective flows of the binary mixture, we use the equations for an incompressible fluid which obtained for the first time in [6] from the convection equations in the Boussinesq approximation:

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\nabla)\boldsymbol{v} = -\nabla p + \Delta \boldsymbol{v} + \frac{\mathrm{R}\,H}{\mathrm{Pr}}\,(T - C)\boldsymbol{\gamma}; \tag{2.1}$$

$$\frac{\partial T}{\partial t} + (\boldsymbol{v}\nabla)T = \frac{1}{\mathrm{Pr}}\,\Delta T, \qquad \operatorname{div} \boldsymbol{v} = 0; \tag{2.2}$$

$$\frac{\partial C}{\partial t} + (\boldsymbol{v}\nabla)C = \frac{1}{\mathrm{Sc}}\,(\Delta C + \varepsilon_0 \Delta T). \tag{2.3}$$

Here $\boldsymbol{v}$, T, p, and C are the dimensionless fields of velocity, temperature, pressure, and admixture concentration. The equations were made dimensionless by using the channel half-thickness d as the unit length, d^2/ν as the unit time, ν/d as the unit velocity, θ as the unit temperature, $\theta\beta_t/\beta_c$ as the unit concentration, and $\rho\nu^2/d^2$ as the unit pressure (ν and β_t are the kinematic viscosity and the temperature coefficient of volume expansion, respectively, θ is the characteristic temperature drop, and ρ is the average fluid density). Equations (2.1)–(2.3) were derived using the equation of state

$$\rho = \rho_0(1 - \beta_t T + \beta_c C).$$

The effects caused by the presence of an admixture in the fluid are characterized by the empirical constants D, β_c, and α. The coefficient β_c describes the dependence of the density of a fluid element on the concentration. Since the effect of a heavy admixture on convection is taken into account, in the case considered, $\beta_c > 0$. The coefficient α characterizes thermal diffusion in the binary fluids, and D is the diffusion coefficient. In the approximation (2.1)–(2.3), it is assumed that the mass and heat fluxes depend on the concentration and temperature gradients as follows:

$$\boldsymbol{J} = -\rho D(\nabla C + \alpha \nabla T), \qquad \boldsymbol{q} = -\varkappa \nabla T$$

($\varkappa$ is the thermal conductivity). System (2.1)–(2.3) contains dimensionless parameters, in particular, the Prandtl number $\mathrm{Pr} = \nu/\chi$, the Schmidt number $\mathrm{Sc} = \nu/D$, and the Rayleigh number $\mathrm{R} = g\beta_t\theta d^3/(\nu\chi)$. The problem also contains additional dimensionless quantities: the height of the channel H and the parameter $\varepsilon_0 = \alpha\beta_c/\beta_t$

which characterizes the thermal diffusion phenomenon in the mixture ($\alpha = k_T/T$, where k_T is the thermal diffusion relation).

In the calculations on the vertical boundaries of the channels, for the velocity we use the slip condition $\boldsymbol{v}|_\Gamma = 0$. Since the walls of the channels are perfectly heat conducting, the temperatures on the vertical boundaries of the calculated perturbation region should be equal to zero. The solid boundaries of the channels will be considered impermeable to material; therefore, the normal component of the diffusion flux density should be equal to zero. Making the expression for the diffusion flux density dimensionless, we obtain the boundary condition

$$\frac{\partial C}{\partial \boldsymbol{n}} + \varepsilon_0 \frac{\partial T}{\partial \boldsymbol{n}} = 0. \tag{2.4}$$

In addition, a necessary condition is the condition of zero fluid flux through the section of both channels:

$$\iint\limits_S (v_z^{(1)} + v_z^{(2)})\, dx\, dy = 0.$$

For certain values of the temperature and concentration gradients in the fluid, a state of mechanical equilibrium is possible in which the fluid is motionless (the velocity is equal to zero and does not vary with time):

$$\partial/\partial t = 0, \quad \boldsymbol{v} = 0, \quad p = p_0, \quad T = T_0, \quad C = C_0.$$

Here T_0, p_0, and C_0 are the equilibrium fields of temperature, pressure, and admixture concentration. Subjecting Eq. (2.1) to the rot operation, for the binary mixture in mechanical equilibrium, we obtain the system of equations

$$(\nabla T_0 - \nabla C_0) \times \boldsymbol{\gamma} = 0, \qquad \Delta T_0 = 0, \qquad \Delta C_0 = 0. \tag{2.5}$$

We consider the case where the equilibrium temperature gradient corresponds to the linear distribution $T_0 = -z/H$ (heating from below). In this case, the Laplace equation for temperature in (2.5) is satisfied identically, and the Laplace equation for concentration allows one to determine the equilibrium distribution of the admixture in the channels due to thermal diffusion. Using the boundary condition (2.4) on the upper and lower boundaries of the channels, we obtain the linear vertical distribution

$$C_0 = \varepsilon_0 z/H.$$

The admixture concentration increases or decreases with changing height, depending on the sign of the thermal diffusion parameter in the equilibrium state.

3. Solution Procedure. The great height of the channels allows Eq. (2.1) to be simplified as follows. Assuming that the approximation of straight-line trajectories is valid, we have $\boldsymbol{v}(0, 0, u)$, where $u(x, y, t)$ is the velocity along the z axis. To eliminate the pressure gradient from Eq. (2.1), we integrate it over a closed contour along the channels. As a result, the Navier–Stokes equation becomes

$$\frac{\partial u}{\partial t} = \Delta u + \frac{\mathrm{R}}{2\mathrm{Pr}}\Big(\int\limits_0^H (T_1 - T_2)\, dz - \int\limits_0^H (C_1 - C_2)\, dz\Big), \tag{3.1}$$

where the subscripts 1 and 2 correspond to the left and right channels. We linearize the Navier–Stokes equations in the approximation of straight-line trajectories. Equations (3.1), (2.2), and (2.3) are solved using the Galerkin method. The velocity and temperature are approximated across the channels so as to satisfy the boundary conditions

$$v_z = u(z, t) \sin(\pi x/2) \cos(\pi y/2), \qquad T = \theta(z, t) \sin(\pi x/2) \cos(\pi y/2). \tag{3.2}$$

The presence of boundary condition (2.4) allows us to transform to the variable $F = C + \varepsilon_0 T$. To satisfy the oddness condition of the profile along x and boundary condition (2.4) for the mass flux on the lateral walls, we construct basis functions for F in the form of linear combinations of trigonometric functions. We introduce special functions of the form

$$S_{13}(x) = \sin(\pi x/2) - (1/3) \sin(3\pi x/2), \qquad C_{13}(x) = \cos(\pi x/2) + (1/3) \cos(3\pi x/2).$$

In view of the definition of the new special functions, the decomposition for $F(x, y, z, t)$ is written as

$$F = f(z, t) S_{13}(x) C_{13}(y). \tag{3.3}$$

Substituting decompositions (3.2) and (3.3) into Eqs. (2.2), (2.3), and (3.1), we perform averaging with the corresponding weight along the section of the channels. In the stationary approximation for the amplitudes $u(z)$, $\theta(z)$, and $f(z)$, we obtain the system of ordinary differential equations

$$\frac{1}{\mathrm{Pr}}\,\theta''_{1,2} \mp \frac{64}{9\pi^2}\,u\theta'_{1,2} - \frac{\pi^2}{2\mathrm{Pr}}\,\theta_{1,2} \pm \frac{u}{H} = 0; \tag{3.4}$$

$$f''_{1,2} \mp \frac{4.28u\mathrm{Sc}}{\pi^2}\,f'_{1,2} - \frac{9\pi^2}{10}\,f_{1,2} = \frac{81\varepsilon_0\mathrm{Sc}}{100\mathrm{Pr}}\Big(\frac{\pi^2}{2}\,\theta_{1,2} - \theta''_{1,2}\Big); \tag{3.5}$$

$$\frac{\pi^2}{2}\,u - \frac{\mathrm{R}}{2\mathrm{Pr}}\Big((1+\varepsilon_0)\int_0^H(\theta_1-\theta_2)\,dz - \int_0^H(f_1-f_2)\,dz\Big) = 0. \tag{3.6}$$

For system (3.4)–(3.6), the boundary conditions on the upper and lower ends of the channels are formulated as

$$z = 0, H\colon\quad \theta_1 = \theta_2, \quad f_1 = f_2, \quad \theta'_1 = -\theta'_2, \quad f'_1 = -f'_2. \tag{3.7}$$

Conditions (3.7) imply that the bridges between the channels are considered short and the temperature and concentration of the fluid passing through the connecting regions do not change upon interaction with the walls.

4. Temperature and Admixture Concentration Distributions along the Height. We seek a solution of the inhomogeneous differential equations with constant coefficients (3.4)–(3.6) in exponential form. The constants related to the roots of the characteristic equations will be denoted by

$$d = \frac{bu\mathrm{Pr}+q}{2}, \quad \gamma = \frac{bu\mathrm{Pr}-q}{2}, \quad \delta = \frac{su\mathrm{Sc}+l}{2}, \quad \varepsilon = \frac{su\mathrm{Sc}-l}{2}.$$

Here $q = \sqrt{(bu\mathrm{Pr})^2+a}$, $a = \pi^2/2$, $l = \sqrt{(su\mathrm{Sc})^2+g}$, $g = 9\pi^2/10$, $b = 32/(9\pi^2)$, and $s = 0.434/\pi^2$.

Joining the common solutions for the left and right channels depending on the vertical coordinate z for θ and f, we obtain the following distributions:

$$\theta_1 = \frac{\mathrm{Pr}\,u}{aqH}\Big(\frac{2\gamma\,\mathrm{e}^{2dz}}{\mathrm{e}^{2dH}+1} - \frac{2d\,\mathrm{e}^{2\gamma z}}{\mathrm{e}^{2\gamma H}+1} + q\Big); \tag{4.1}$$

$$f_1 = \frac{A\,\mathrm{e}^{2\delta z}}{\mathrm{e}^{2\delta H}+1} + \frac{B\,\mathrm{e}^{2\varepsilon z}}{\mathrm{e}^{2\varepsilon H}+1} + \frac{\xi\,\mathrm{e}^{2dz}}{\mathrm{e}^{2dH}+1} + \frac{\eta\,\mathrm{e}^{2\gamma z}}{\mathrm{e}^{2\gamma H}+1} + \chi; \tag{4.2}$$

$$\theta_2 = -\theta_1(H-z), \qquad f_2 = -f_1(H-z);$$

$$\frac{\mathrm{R}\,(1+\varepsilon_0)u}{Ha^2q}\,[4d^2\tanh\,(\gamma H) - 4\gamma^2\tanh\,(dH) + Haq]$$

$$-\frac{\mathrm{R}}{2\mathrm{Pr}}\Big(\frac{A}{\delta}\tanh\,(2\delta H) + \frac{B}{\varepsilon}\tanh\,(2\varepsilon H) + \frac{\xi}{d}\tanh\,(2dH) + \frac{\eta}{\gamma}\tanh\,(2\gamma H) + 2\chi H\Big) - au = 0. \tag{4.3}$$

In (4.1)–(4.3), the constants have the following values:

$$\xi = \frac{0.81\varepsilon_0\gamma\mathrm{Sc}u(\pi^2-8d^2)}{aqH(4d-4sd\mathrm{Sc}u-g)}, \quad \eta = -\frac{0.81\varepsilon_0 d\mathrm{Sc}u(\pi^2-8\gamma^2)}{aqH(4\gamma^2-4s\gamma\mathrm{Sc}u-g)}, \quad \chi = -\frac{0.45\varepsilon_0\mathrm{Sc}u}{aH}.$$

The amplitudes A and B are expressed in terms of the constants ξ, η, and χ:

$$A = [\eta(\varepsilon-\gamma) + \xi(\varepsilon-d) + 2\chi\varepsilon]/l, \qquad B = -[\eta(\delta-\gamma) + \xi(\delta-d) + 2\chi\delta]/l.$$

The dashed curve in Fig. 2 shows the temperature dependence on the vertical coordinate. It is evident that, at the entrance into the channel, the fluid temperature changes rapidly and then reaches the asymptotic value. Figure 3 shows the admixture concentration distribution along the height for the same parameters. It is evident that, in the case of positive thermal diffusion, there is generally a deficit of the heavy admixture in the region of ascending flow, and in the channel with descending flow, it is present in excess. Thus, a convective loop can be used as a facility to separate mixtures into components. The separation of the mixtures occurs against the background of convective transfer, i.e., much more rapidly than in a static thermal diffusion column.

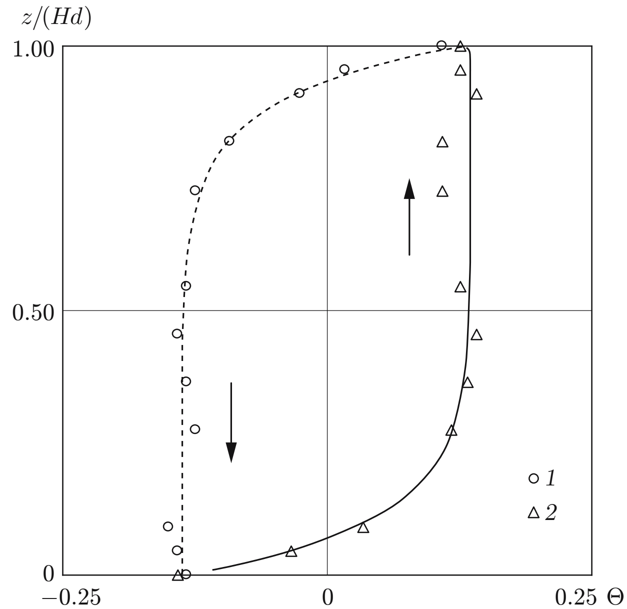

Fig. 2

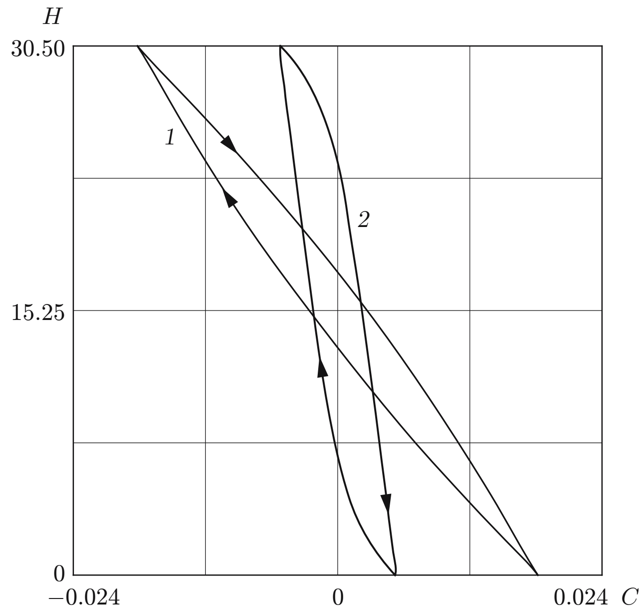

Fig. 3

Fig. 2. Vertical temperature distribution along the axes of the right (1) and left (2) channels for $\varepsilon_0 = 0.02$, $H = 30.5$, $\mathrm{Sc} = 1000$, $\mathrm{Pr} = 7$, and $\mu = 1.2$: points refer experimental data, and curves refer to calculations of stationary flow; arrows show the flow direction in the channels.

Fig. 3. Vertical distribution of admixture concentration along the axes of the right and left channels for flow velocities $u = 3$ (1) and 1.5 (2); arrows show the flow direction in the channels.

5. Finite Amplitude Flows. Formula (4.3) relates the velocity amplitude u to the Rayleigh number. For a homogeneous fluid in the limiting case $u \to 0$ for $\varepsilon_0 = 0$, we obtain the expression defining the stability boundary of the equilibrium with respect to monotonic perturbations as a function of the channel height:

$$\mathrm{R}_c = \pi^4/[4(1 - z_1^{-1} \tanh z_1)]. \qquad (5.1)$$

Here $z_1 = \pi H/(2\sqrt{2}\,)$. From expression (5.1), it follows that, as the channel length increases, the critical Rayleigh number decreases. In the limit $H \to \infty$, from relation (5.1) we obtain the critical value of the Rayleigh number for infinite channels $\mathrm{R}_c = \pi^4/4$ [1]. In the case of arbitrary values of the thermal diffusion parameter and the Schmidt and Prandtl numbers, the critical Rayleigh number is expressed as

$$\mathrm{R}_c = \frac{\pi^4}{4}\Big[(1+\varepsilon_0)\Big(1 - \frac{1}{z_1}\tanh z_1\Big) + \frac{\varepsilon_0 \mathrm{Sc}}{\mathrm{Pr}}\Big(0.45 - \frac{1}{z_2}\tanh z_2\Big)\Big]^{-1}. \qquad (5.2)$$

Here the argument z_2 is calculated by the formula $z_2 = 3\sqrt{10}\,\pi H/20$. In the presence of thermal diffusion, the convection threshold depends on the Prandtl number. In the case of negative values of the thermal diffusion parameter, the critical Rayleigh number decreases with increasing Prandtl number. In the case of a positive thermal diffusion effect, the threshold Rayleigh number increases with increasing Prandtl number, and, in the limit $\mathrm{Pr} \to \infty$, it reaches the asymptotic value $\mathrm{R}_c \approx 24.5$. Using expression (4.3), it is possible to calculate the flow amplitude for arbitrary values of the Rayleigh number. For comparison with the experiment, we use the supercriticality parameter $\mu = \mathrm{R}\,/\,\mathrm{R}_c$. The nature of convection initiation can be determined from the amplitude curves in Fig. 4. An analysis of curve 1 plotted for a positive value of ε_0 shows that, in the case of the normal Soret effect, stationary flows branch off softly. In the case of the anomalous Soret effect (curve 2), rigid initiation of monotonic convection is possible. For a one-component fluid in the limit $\varepsilon_0 = 0$, expression (5.2) coincides with the expression obtained in [2] (the dashed curve in Fig. 4). From Fig. 4, it follows that, for small values of the supercriticality parameter, the presence of admixture has a significant effect on the flow. As the Rayleigh number increases, the intensity of flow increases and the admixture spreads over the channels.

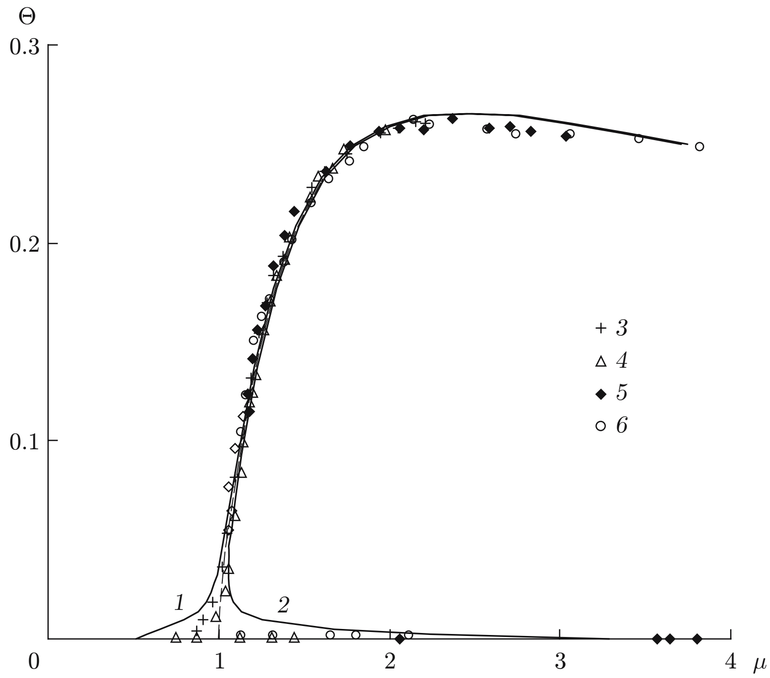

Fig. 4. Amplitude curves of stationary flows: the solid curves are theoretical curves for $H = 30.5$, $\mathrm{Sc} = 700$, $\mathrm{Pr} = 7$, and $\varepsilon_0 = 0.02$ (1) and -0.015 (2); the dashed curve refers to a one-component fluid ($\varepsilon_0 = 0$); points refer to experiment with water (3), a 5% solution of C_2H_5OH in water (4), 15% solution of Na_2SO_4 in water (5), a solution of CCl_4 in decane (various concentrations) (6)).

6. Experiment. The experimental setup (see Fig. 1) consists of a metal rod and is equipped with massive isothermal heat-transfer devices. Water circulated from a jet thermostat through the upper heat-transfer device. The lower heat-transfer device is equipped with an electric heater with digital control by a Termodat T16E2 PID-controller. The rod has two longitudinal parallel channels of square section of width $2d = 3.2$ mm; the channels are connected at the top and bottom by bridges of the same profile. The height of the vertical channels is $h = 50$ mm. On one side, the channels were covered with a transparent Plexiglas plate. The flow rate was recorded by a differential thermocouple (see Fig. 1a) with an electrode diameter of 0.1 mm, placed at the center of the channels. Each thermocouple junction is 1.5 mm long and reaches the center of the channel; therefore, its presence leads to averaging of the mixture temperature in the cross-section of the channel. The theoretical calculations and thermocouple readings were compared using an empirical averaging coefficient. In some experiments in the channels, additional 11 thermocouples were placed to measure the temperature distribution along the height. The EMF of the thermocouples was measured by a Termodat T29BM1 digital device (see Fig. 1c). The measuring devices were connected to the USB port of the computer through a RS232/RS485 converter and were inquired using the Termodat 7.29 code. The temperature measurement technique used in the experiment is described in detail in [7]. As the measure of the flow intensity we used the dimensionless quantity $\Theta = |\zeta|/\Delta T$, where ζ is the thermocouple reading (see Fig. 1a) and ΔT is the vertical temperature difference between the heat-transfer devices.

The experiments were performed with aqueous solutions of sodium sulfate Na_2SO_4 and ethyl alcohol. Solutions of sodium sulfate in water are characterized by large positive Soret coefficients (Sr) [8]. Thus, for a solution with an average concentration $C_0 = 0.157$, the Soret and diffusion coefficients are $\mathrm{Sr} = 8.9 \cdot 10^{-3}\ \mathrm{K}^{-1}$ and $D = 0.6 \cdot 10^{-5}\ \mathrm{cm}^2/\mathrm{sec}$, respectively. The dimensionless parameters describing this mixture have values $\mathrm{Pr} = 8.5$ and $\mathrm{Sc} = 2.1 \cdot 10^3$, and the thermal diffusion parameter is $\varepsilon_0 = \mathrm{Sr}\,\beta_c/\beta_t = 0.36$. Aqueous solutions of ethyl alcohol can have both positive and negative thermal diffusion, depending on the concentration. In the experiment, we used a 5% solution C_2H_5OH with a negative thermal diffusion coefficient $\varepsilon_0 = -0.3$, a Prandtl number $\mathrm{Pr} = 7$, and a Schmidt number $\mathrm{Sc} = 2500$.

Water and aqueous solutions are well suitable for experiments on the setup described here since the critical water temperature difference for equilibrium crisis is 6.3 °C, and the described measuring system allows the thermal

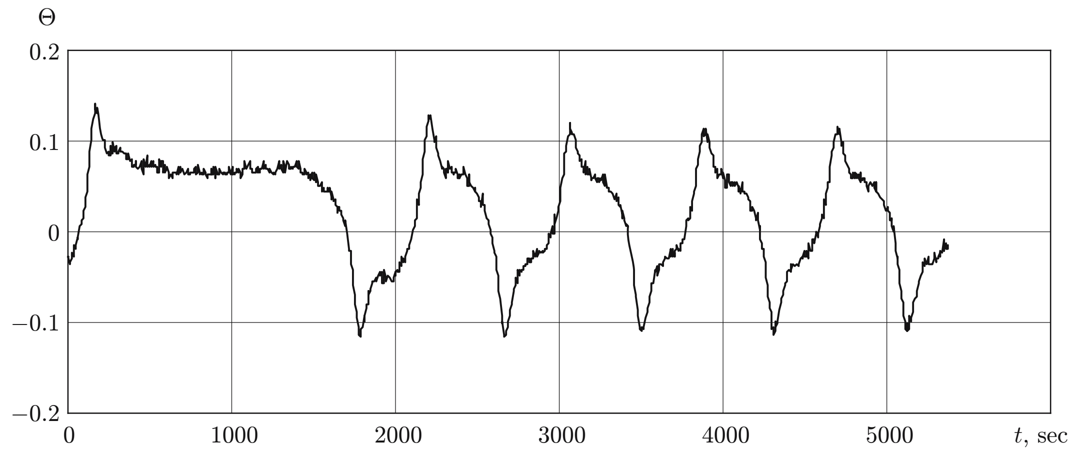

Fig. 5. Periodic oscillations of a binary mixture (a 16% solution of Na_2SO_4 in water) in connected channels in the case of positive thermal diffusion.

Rayleigh number to be changed in small steps. In the experiment, the channels were filled with the fluid studied and the temperature difference between the heat-transfer devices was set to be larger than the critical difference. This resulted in transition from the state of equilibrium to intense convection. At a negative Soret coefficient after the transition process, the thermocouple readings ζ and ΔT always reach stationary values, which were plotted. During the experiment, the temperature difference increased or decreased with a step of 0.1–1.0°C. The experimental deviations of the temperature field were normalized by the theoretical value of Θ at the point of maximum of the curve (see Fig. 4). Figure 2 shows the experimental temperature distribution along the channels. In Fig. 4, the results of the experiments with water and mixtures are plotted on the amplitude curve. There is good agreement with theory. It is also noted that, regardless of the sign of the thermal diffusion parameter, in most experiments with $\mu > 1.1$, the concentration inhomogeneities are smeared along the channel and do not influence the flow.

However, it should be noted that some of the experimental data differ significantly from the calculation results for stationary flows. For a mixture with normal thermal diffusion (an aqueous solution of sodium sulfate) with $\mu \approx 1.1$, the stationary flow became unstable and periodic oscillations with a period of approximately 0.5 h arose in the channels (Fig. 5). These oscillations are due to the effect of concentration inhomogeneities on the convective flow. The concentration inhomogeneities are generated by thermal diffusion due to horizontal temperature gradients, which, in turn, occur only in convective circulation of the mixture. The nature of these oscillations is discussed in [7] and confirmed by numerical modeling using the finite difference method.

It should also be noted that, in experiments with mixtures, regardless of the sign of the thermal diffusion parameter, the mechanical equilibrium rigidly loses stability with respect to oscillatory perturbations, too. The equilibrium of a mixture with normal thermal diffusion in the supercritical range ($\mu > 1$) is unusual at first glance, since positive thermal diffusion (curve 1 in Fig. 4) reduces the stability threshold and instability should remain monotonic in this case. However, in the experiments, oscillatory instability of the mechanical equilibrium occurs, and in different experiments, equilibrium becomes unstable for different values of the supercriticality parameter. This implies that the existence of the uncontrollable parameter in the experiments — the concentration density gradient directed downward. If the mixture is initially not quite homogeneous, this nonequilibrium concentration gradient results from the hydrodynamic stratification of the inhomogeneities, similar to a poorly mixed syrup gravitating to the bottom of a glass with water. For fluid mixtures ($\mathrm{Sc} \gg \mathrm{Pr} \gg 1$) in infinitely long channels with heat-conducting walls in the presence of a vertical concentration gradient, the frequency of neutral oscillations can be written in dimensional form [3]

$$\nu = \pi\chi\sqrt{\mu_{\mathrm{os}} - 1}/(4d^2), \tag{6.1}$$

where $\mu_{\mathrm{os}} = \mathrm{R_{os}}/\mathrm{R}_c$ is the supercriticality at which oscillatory instability occurs. The validity of the above expression has been confirmed experimentally [4]. At the same time, the formula for the neutral curve of the oscillatory instability leads to the following expression for the heavy-admixture concentration gradient:

$$\nabla C = (\beta_t/\beta_c)\nabla T_c\,(\mu_{\mathrm{os}} - 1). \tag{6.2}$$

Combining relations (6.1) and (6.2), we obtain the expression

$$\nabla C = \frac{\beta_t}{\beta_c}\, \nabla T_c\, \frac{16\nu^2 d^4}{\pi^2 \chi^2}.$$

In view of the frequency of the transient oscillations, the critical temperature gradient, the geometrical dimensions of the channels and the thermal properties of the mixture, the concentration gradient can be estimated as $5 \cdot 10^{-3}$–$5 \cdot 10^{-2}$ cm^{-1}. Hence, to obtain oscillatory initiation of convection with a hysteresis in experiments, a small inhomogeneity of the mixture along the vertical is sufficient (a few percent or fractions of percent of the mass-averaged concentration). This gradient is not equilibrium, and the time of diffusion equalization is hundreds of hours: $(Hd)^2/(\pi^2 D) \approx 10^2$ h.

Conclusions. The effect exerted on thermal convection by an admixture in a fluid in connected channels of finite height was studied theoretically and experimentally. The stability boundaries of the mechanical equilibrium for monotonic perturbations were found experimentally and the nature of convection initiation in the case of positive and negative thermal diffusion was determined. The form of stationary and oscillatory flows for finite supercriticality values was studied. The results were confirmed experimentally.

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